75th MORSS CD Cover Slide

712CD

For office use only 41205

UNCLASSIFIED DISCLOSURE FORM CD Presentation

12-14 June 2007, at US Naval Academy, Annapolis, MD

Please complete this form 712CD as your cover page to your electronic briefing submission to the MORSS CD. Do not fax to the MORS office.

Author Request (To be completed by applicant) - The following author(s) request authority to disclose the following presentation in the MORSS Final Report, for inclusion on the MORSS CD and/or posting on the MORS web site.

Name of Principal Author and all other author(s):

Dr. Roy E. Rice

Principal Author's Organization and address:

Teledyne Brown Engineering
300 Sparkman Dr.
Huntsville, AL 35805

Phone: 256-726-2038

Fax: 256-726-3434

Email: roy.rice@tbe.com

Original title on 712 A/B: **Quantifying "Persistence" in the Context of Find-Fix-Finish (FFF)**

Revised title:____

Presented in (input and Bold one): (WG **5**, CG___, Special Session ___, Poster, Demo, or Tutorial):

This presentation is believed to be:
UNCLASSIFIED AND APPROVED FOR PUBLIC RELEASE

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
01 JUN 2007	**N/A**	**-**

4. TITLE AND SUBTITLE
Quantifying Persistence In The Context Of Find, Fix, Finish

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Teledyne Brown Engineering 300 Sparkman Dr. Huntsville, AL 35805

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADM202526. Military Operations Research Society Symposium (75th) Held in Annapolis, Maryland on June 12-14, 2007., The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**UU**	**31**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

TELEDYNE
BROWN ENGINEERING, INC.
A Teledyne Technologies Company

QUANTIFYING “PERSISTENCE” IN THE CONTEXT OF FIND, FIX, FINISH

Dr. Roy Rice

13-15 June 2007

Outline

- Set the Stage
- Define the problem
- Probability derivation
- Limits
- Derivatives to show rate of change
- Persistent ISR Ratio (PIR)
- Examples

Set the Stage

- *"Maintaining* ***persistent ISR*** *around the globe would allow the military to continue to function as a 'strategically relevant, continental United States-based projection force,' Bair said during the Defense News Media Group conference, ISR Integration 2003: The Net-Centric Vision, in Arlington, Va."* Ref[1]. Edward Bair is the Army program executive officer for intelligence, electronic warfare and sensors.

Set the Stage (cont.)

- *"We were trying to craft more of a story and a message that says we're moving away from, say, the reconnaissance paradigm to that* ***persistent surveillance*** *paradigm and let's look at what we're buying and see if that really does accomplish where we're trying to go."* Ref [2] - Kevin Meiners; the director of intelligence strategies, technologies and assessments, Office of the Deputy Undersecretary of Defense for Intelligence and Warfighting Support.

Set the Stage (cont.)

- *"We need long-term investment in **persistent ISR** capability with assured electromagnetic spectrum access utilizing up-to-date collection technologies to **find, track and interdict** mobile and technologically competent terrorist groups and platforms operating with the vast regions of Africa and Europe, including both air and maritime environments."* Ref[3]– General James L. Jones, USMC, Commander United States European Command, before the House Armed Services Committee on 8 March 2006.

TELEDYNE
BROWN ENGINEERING, INC.
A Teledyne Technologies Company

Set the Stage (cont.)

- *"Precision operations are intelligence-driven. As noted above, we need to rebalance our **'find, fix, finish'** targeting cycle."* Ref[4] – General John P. Abizaid, USA, Commander, United States Central Command, before the House Armed Service Committee on 15 March 2006.

Set the Stage (cont.)

- In an August 10, 2006, interview in Secretary Rumsfeld's office, the Secretary said, *"I was asked that when I was up at the confirmation hearings in January of '01, and I said intelligence. And if you think about this department, we have just enormous capability to **finish**. If you use the phrase "**find, fix and finish**," we can **finish** something if we can **find** it and **fix it in time** and location. The problem is **finding** it. And you can find big armies and big navies and big air forces, and we've gotten quite good at that in this department. It is a whale of a lot harder to deal with a network, with individuals, with people that don't wear uniforms, with people that mix among civilians and hide among innocent people.*" Ref[5]

TELEDYNE BROWN ENGINEERING, INC.
A Teledyne Technologies Company

Set the Stage (cont.)

- And, finally, in a news briefing on 12 January 2006 with Secretary Rumsfeld and Chairman General Peter Pace, Secretary Rumsfeld said, *"And the reality that this department has responsibilities to **find and to fix and to finish** -- to use the phrase -- in terms of dealing with threats and enemies to this country, and a recognition that we have a great deal of ability to fix and -- correction, to **finish** -- in an operation and much less ability to **find** and **fix**, and the importance of seeing that our department over time recognizes that imbalance and does everything humanly possible to see that we find ways to link -- we are the biggest user of intelligence -- this department is -- and we need to see that there is an intimate relationship in proximity and time between intelligence and operations."* Ref[6]

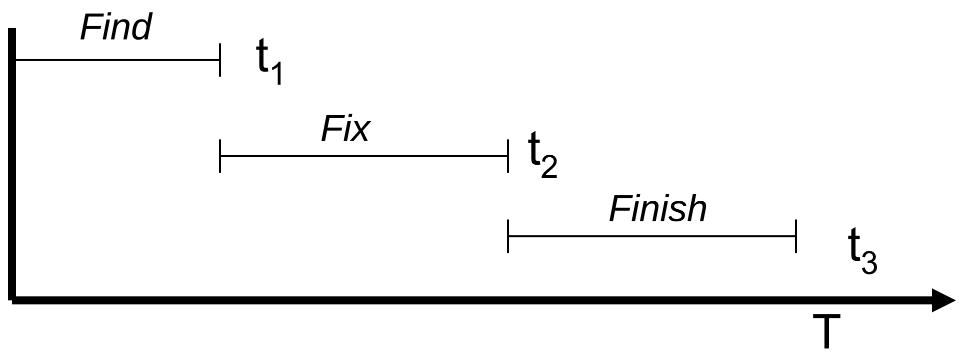
Set the Stage

- Combatant Commands refer to Kill Chain as F-F-F (F3)
  - Find, fix, finish
  - Can be further decomposed into F2T2EA
    - Find, fix, track, target, engage, assess
- From OEG Report on *Search Theory*
  - Prob (event in time t) = 1- $e^{-\lambda t}$
    - Where λ is the rate and t is the time to accomplish the event
    - Density function is $f(t) = \lambda e^{-\lambda t}$
  - We'll assume this density applies to each of the events or phases – Find-Fix-Finish

TELEDYNE
BROWN ENGINEERING, INC.
A Teledyne Technologies Company

Problem Set-up

Borrowing from Reliability Theory of Redundant Systems and Bayes' Theorem:



t_1 = time to Find

t_2 = time to Fix

t_3 = time to Finish

- $P[(t_1 \le T) \cap (t_2 \le T - t_1) \cap (t_3 \le T - t_1 - t_2)] = P(T)$

=

$$\int_0^T \lambda_1 e^{-\lambda_1 t_1} \int_0^{T-t_1} \lambda_2 e^{-\lambda_2 t_2} \int_0^{T-t_1-t_2} \lambda_3 e^{-\lambda_3 t_3} dt_3 dt_2 dt_1$$

* Assume the λ_is are unequal…otherwise it's trivial

$$= 1 - e^{-\lambda_1 T}\left(\frac{\lambda_2\lambda_3}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}\right) - e^{-\lambda_2 T}\left(\frac{\lambda_1\lambda_3}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}\right) - e^{-\lambda_3 T}\left(\frac{\lambda_1\lambda_2}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}\right)$$

TELEDYNE BROWN ENGINEERING, INC.
A Teledyne Technologies Company

In general:

$$P(T) = 1 - \sum_{j} \frac{\left[e^{-\lambda_j T} * \left(\prod_{i \neq j} \lambda_i \right) \right]}{\prod_{i \neq j} \left(\lambda_j - \lambda_i \right)}$$

Problem Solution

- Let $\theta_i = 1/\lambda_i$

  = mean-time-to-accomplish event *i*
- Then, P(T) = P

Can be rewritten as:

$$P(T) = 1 - e^{-\frac{T}{\theta_1}}\left(\frac{\theta_1^2}{(\theta_1 - \theta_2)(\theta_1 - \theta_3)}\right) - e^{-\frac{T}{\theta_2}}\left(\frac{\theta_2^2}{(\theta_2 - \theta_1)(\theta_2 - \theta_3)}\right) - e^{-\frac{T}{\theta_3}}\left(\frac{\theta_3^2}{(\theta_3 - \theta_1)(\theta_3 - \theta_2)}\right)$$

In general:

$$P(T) = 1 - \sum_{j} \frac{\left[e^{-T/\theta_j} * \theta_j^2\right]}{\prod_{i \neq j} \left(\theta_j - \theta_i\right)}$$

TELEDYNE BROWN ENGINEERING, INC.
A Teledyne Technologies Company

Density Function for T

dP/dT

$$\left.dP\middle/dT\right. = \left(\prod_i \lambda_i\right) * \sum_j \frac{\left[e^{-\lambda_j T}\right]}{\prod_{i \neq j} (\lambda_j - \lambda_i)}$$

…or…

$$\left.dP\middle/dT\right. = \sum_j \frac{\left[e^{-T/\theta_j} * \theta_j\right]}{\prod_{i \neq j} (\theta_j - \theta_i)}$$

Limits

$\theta_1 \to 0$

$$P = 1 - e^{-\frac{T}{\theta_2}}\left(\frac{\theta_2}{\theta_2 - \theta_3}\right) - e^{-\frac{T}{\theta_3}}\left(\frac{\theta_3}{\theta_3 - \theta_2}\right)$$

$\theta_2 \to 0$

$$P = 1 - e^{-\frac{T}{\theta_1}}\left(\frac{\theta_1}{\theta_1 - \theta_3}\right) - e^{-\frac{T}{\theta_3}}\left(\frac{\theta_3}{\theta_3 - \theta_1}\right)$$

$\theta_3 \to 0$

$$P = 1 - e^{-\frac{T}{\theta_1}}\left(\frac{\theta_1}{\theta_1 - \theta_2}\right) - e^{-\frac{T}{\theta_2}}\left(\frac{\theta_2}{\theta_2 - \theta_1}\right)$$

Limits are the *P(T)* calculations given "instantaneous" events

SENSITIVITIES OF θ_i's

- Interested in sensitivity of *P(T)* to changes in θ_i's
- Need to perform trade-off studies
  - Operations
  - Cost

Derivatives of P

$$\frac{\partial P}{\partial \theta_1} = -e^{-\frac{T}{\theta_1}}\left(\frac{\theta_1^2}{(\theta_3-\theta_1)^2(\theta_2-\theta_1)} + \frac{\theta_1^2}{(\theta_2-\theta_1)^2(\theta_3-\theta_1)} + \frac{T}{(\theta_3-\theta_1)(\theta_2-\theta_1)} + \frac{2\theta_1}{(\theta_3-\theta_1)(\theta_2-\theta_1)}\right)$$

$$+e^{-\frac{T}{\theta_2}}\left(\frac{\theta_2^2}{(\theta_3-\theta_2)(\theta_2-\theta_1)^2}\right) - e^{-\frac{T}{\theta_3}}\left(\frac{\theta_3^2}{(\theta_3-\theta_2)(\theta_3-\theta_1)^2}\right)$$

$$\frac{\partial P}{\partial \theta_2} = e^{-\frac{T}{\theta_2}}\left(\frac{\theta_2^2}{(\theta_3-\theta_2)^2(\theta_2-\theta_1)} - \frac{\theta_2^2}{(\theta_2-\theta_1)^2(\theta_3-\theta_2)} + \frac{T}{(\theta_3-\theta_2)(\theta_2-\theta_1)} + \frac{2\theta_2}{(\theta_3-\theta_2)(\theta_2-\theta_1)}\right)$$

$$+e^{-\frac{T}{\theta_1}}\left(\frac{\theta_1^2}{(\theta_3-\theta_1)(\theta_2-\theta_1)^2}\right) - e^{-\frac{T}{\theta_3}}\left(\frac{\theta_3^2}{(\theta_3-\theta_1)(\theta_3-\theta_2)^2}\right)$$

$$\frac{\partial P}{\partial \theta_3} = e^{-\frac{T}{\theta_3}}\left(\frac{\theta_3^2}{(\theta_3-\theta_2)^2(\theta_3-\theta_1)} + \frac{\theta_3^2}{(\theta_3-\theta_1)^2(\theta_3-\theta_2)} - \frac{T}{(\theta_3-\theta_1)(\theta_3-\theta_2)} - \frac{2\theta_3}{(\theta_3-\theta_1)(\theta_3-\theta_2)}\right)$$

$$+e^{-\frac{T}{\theta_1}}\left(\frac{\theta_1^2}{(\theta_2-\theta_1)(\theta_3-\theta_1)^2}\right) - e^{-\frac{T}{\theta_2}}\left(\frac{\theta_2^2}{(\theta_2-\theta_1)(\theta_3-\theta_2)^2}\right)$$

Derivatives of P

- Interesting to note (you can manipulate the previous equations to prove it):
  - If I swap θ_i for θ_j , then the "swapped" partials are equal and the third partial stays the same and P(T) remains the same.
  - If I swap all three θ_is , then all the "swapped" partials are equal and P(T) remains the same.
  - Example, let θ_1 =10, θ_2 =15, θ_3 =50, and T = 60. Then
    - P(T) = 0.4845
    - $\frac{\partial P}{\partial \theta_1}$ = - 0.010007
    - $\frac{\partial P}{\partial \theta_2}$ = - 0.009737
    - $\frac{\partial P}{\partial \theta_3}$ = - 0.006251
  - If I let θ_1 =50, θ_2 =10, θ_3 =15, and hold T = 60. Then
    - P(T) = 0.4845
    - $\frac{\partial P}{\partial \theta_1}$ = - 0.006251
    - $\frac{\partial P}{\partial \theta_2}$ = - 0.010007
    - $\frac{\partial P}{\partial \theta_3}$ = - 0.009737

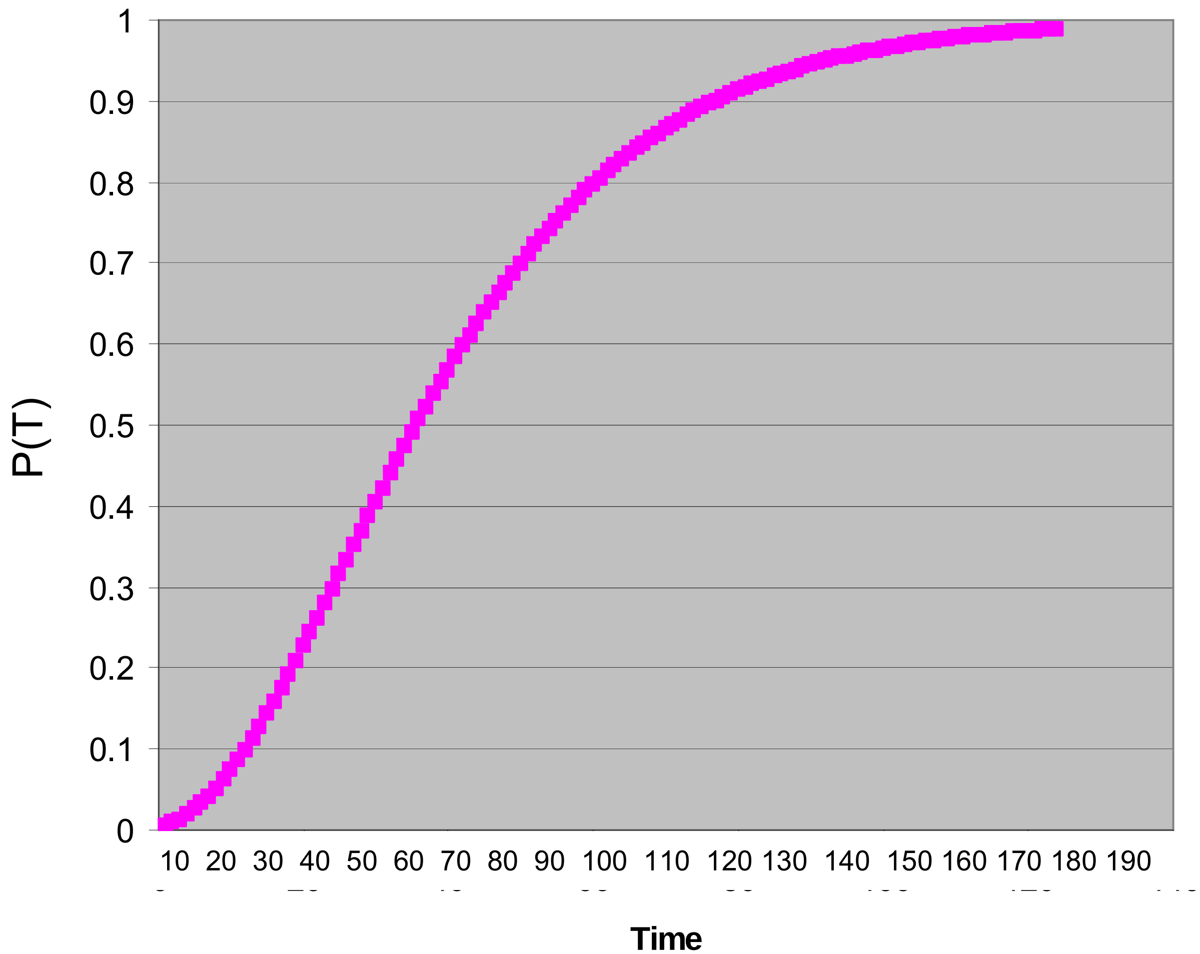
TELEDYNE
BROWN ENGINEERING, INC.
A Teledyne Technologies Company

Persistent ISR Ratio (PIR)

- ISR is mainly concerned with “Find”; although it’s often part of other two.
- Focused on “Find”…but emphasizes others.
- Compares “rates of change” relative to other “F’s”
- If PIR > 1, then reductions in θ_1 result in larger increases in *P(T)*

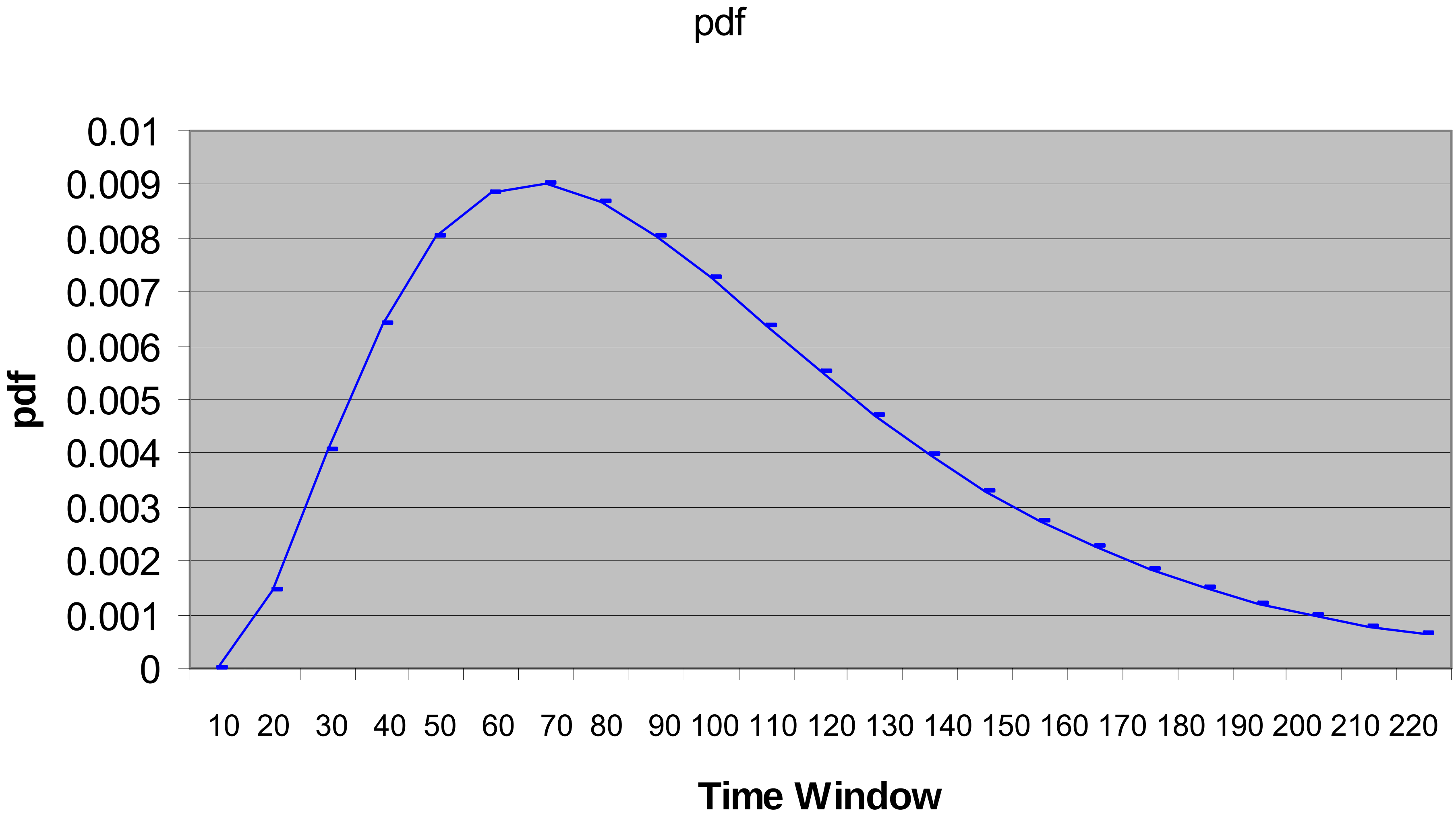
$$\text{PIR} = \min\left\{ \frac{\partial P / \partial \theta_1}{\partial P / \partial \theta_2} \;,\; \frac{\partial P / \partial \theta_1}{\partial P / \partial \theta_3} \right\}$$

P(T) as a Function of Time Window



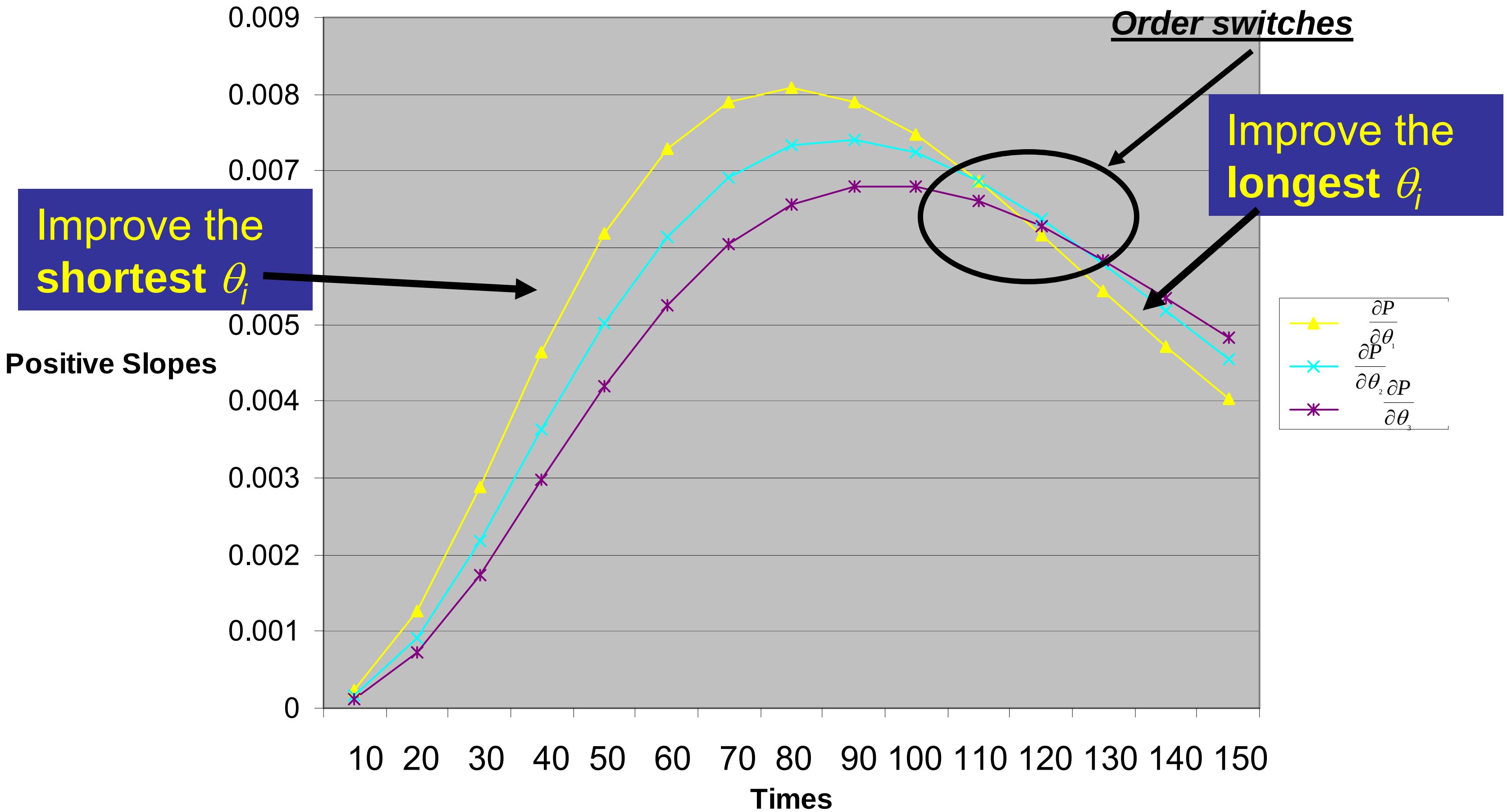
Given: Time varies (X axis) from 10 to 190 time units; θ_i = 20, 30, 40 respectively

Density Function



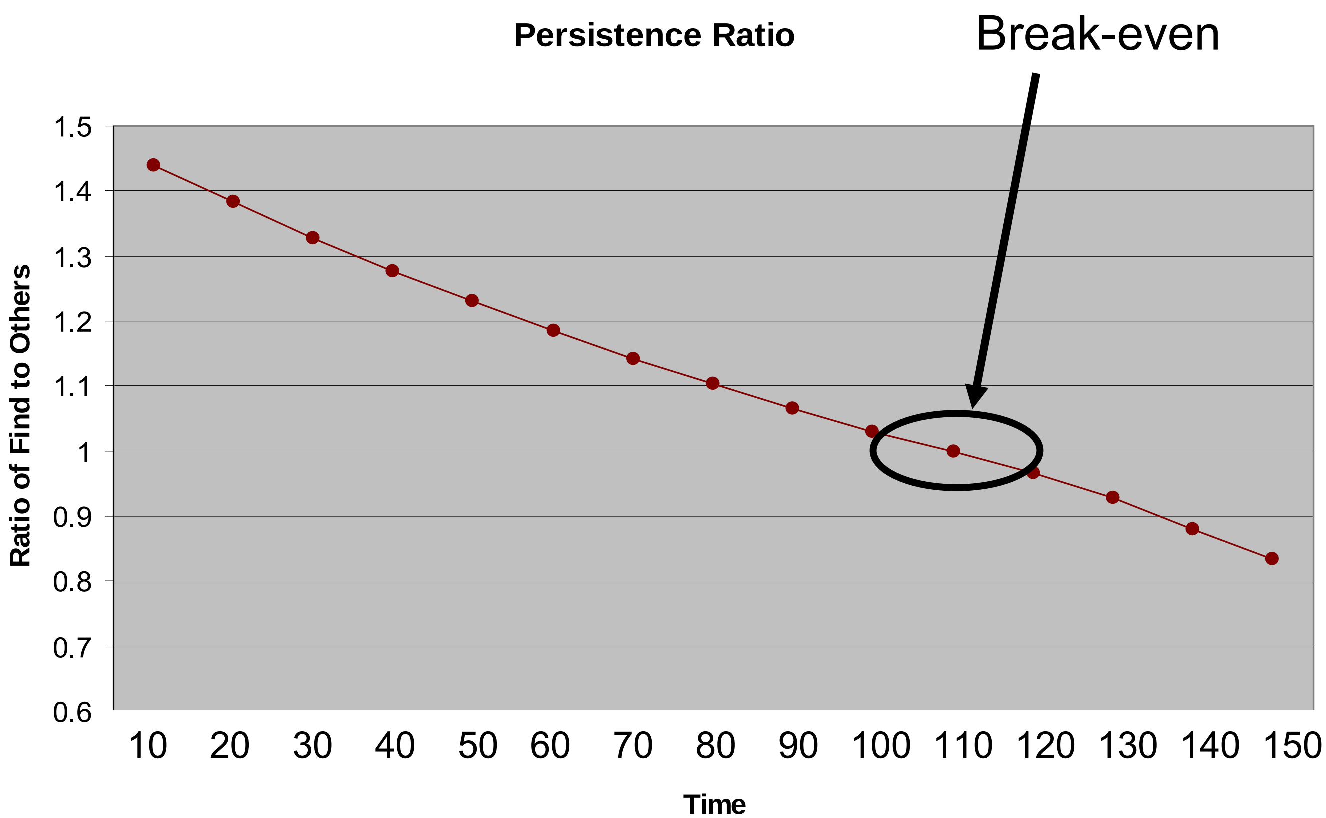
Given: Time varies (X axis) from 10 to 220 time units; θ_i = 20, 30, 40 respectively

Derivatives



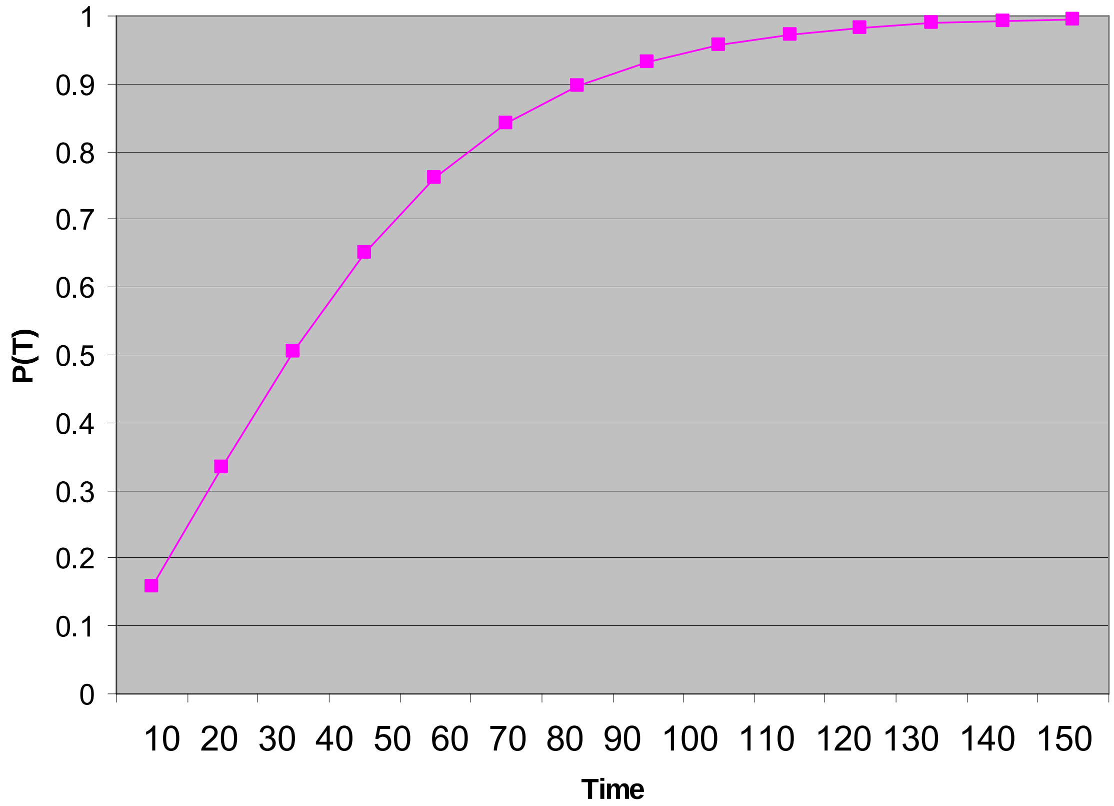
Given: Time varies (X axis) from 10 to 150 time units; θ_i = 20, 30, 40 respectively

Persistent ISR Ratio



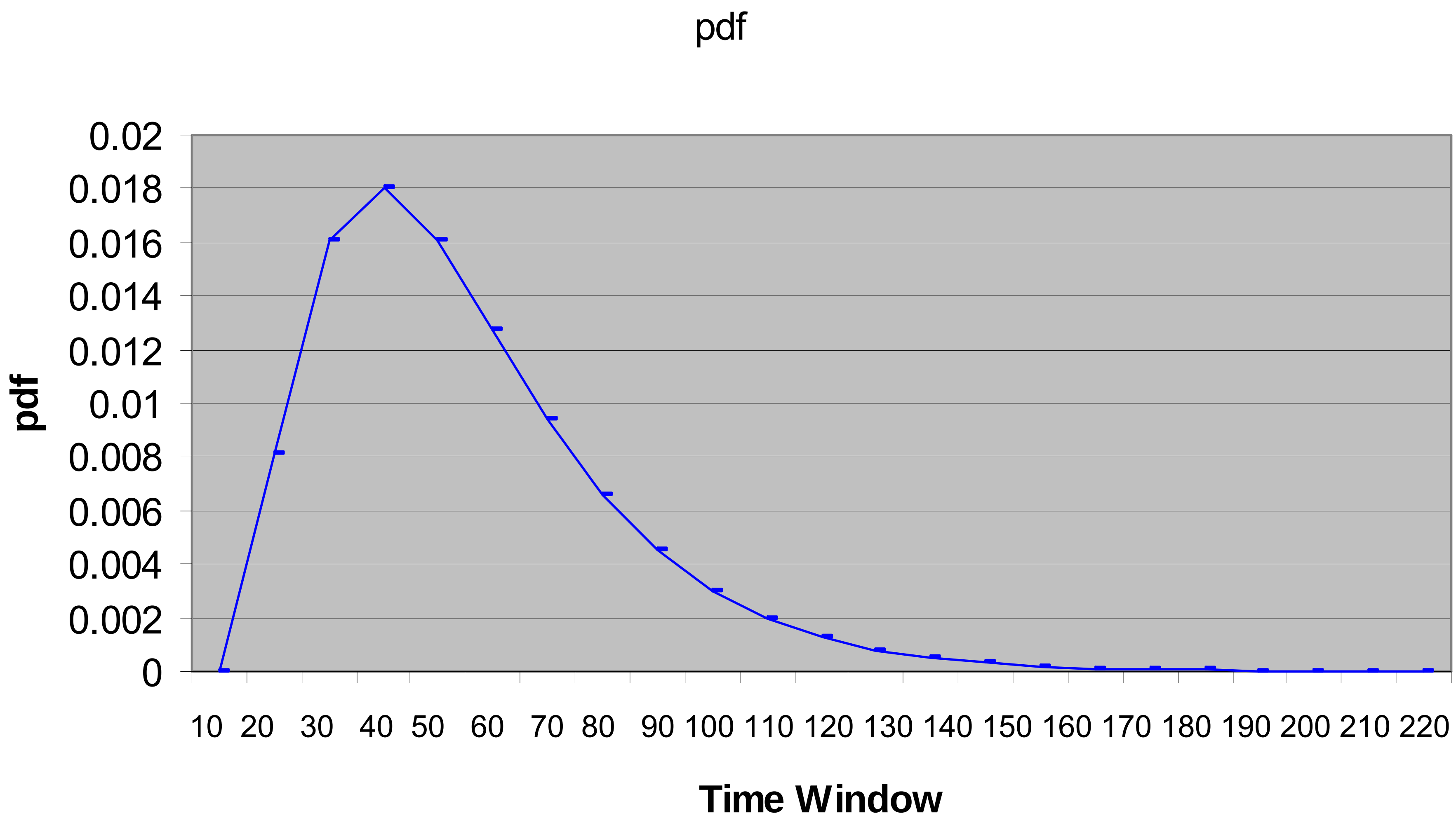
Minimum of Derivative (slope) of “Find” to other Derivatives.

P(T) as a Function of Time Window



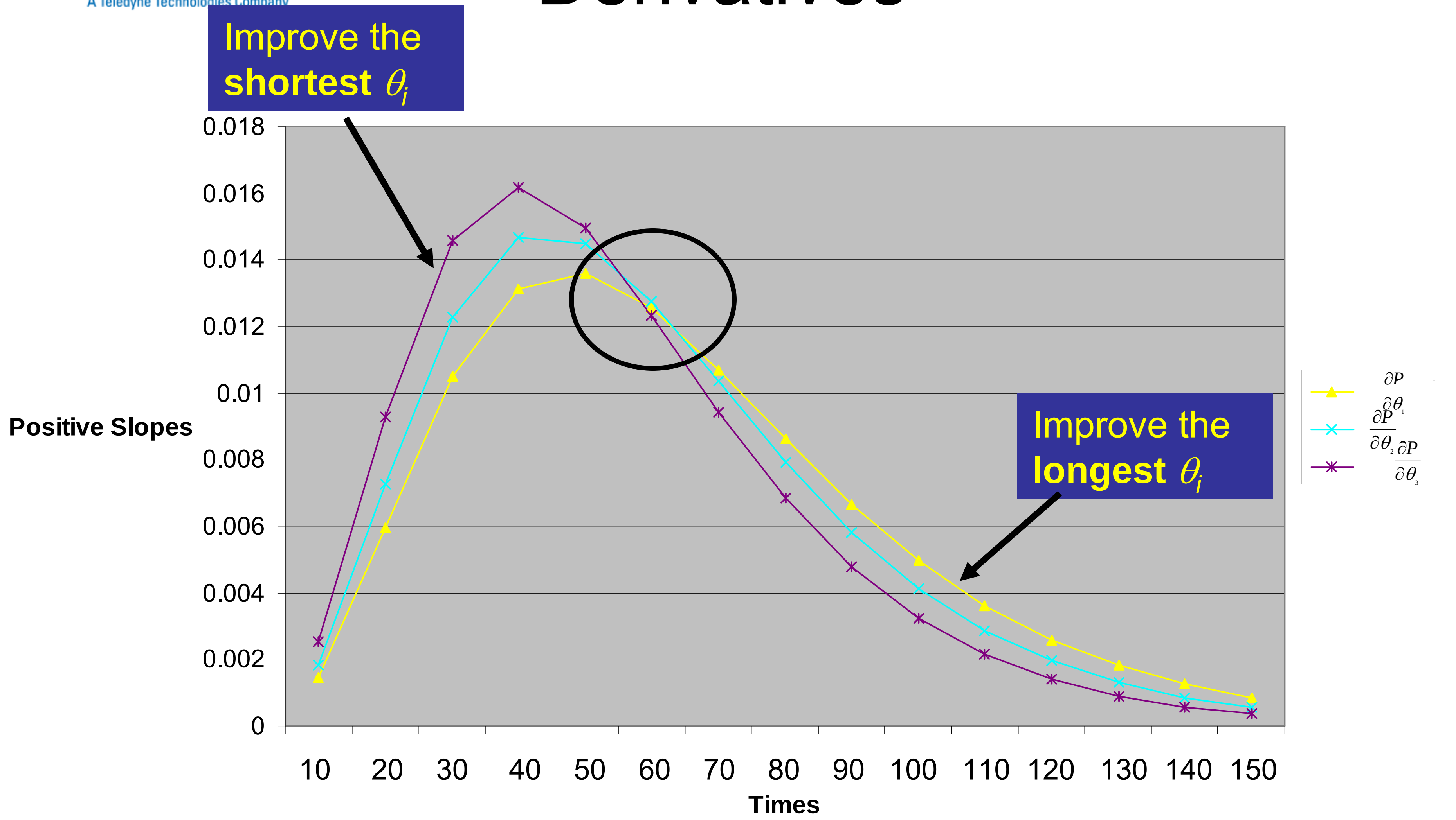
Given: Time varies (X axis) from 10 to 150 time units; θ_i = 20, 15, 10 respectively

Density Function



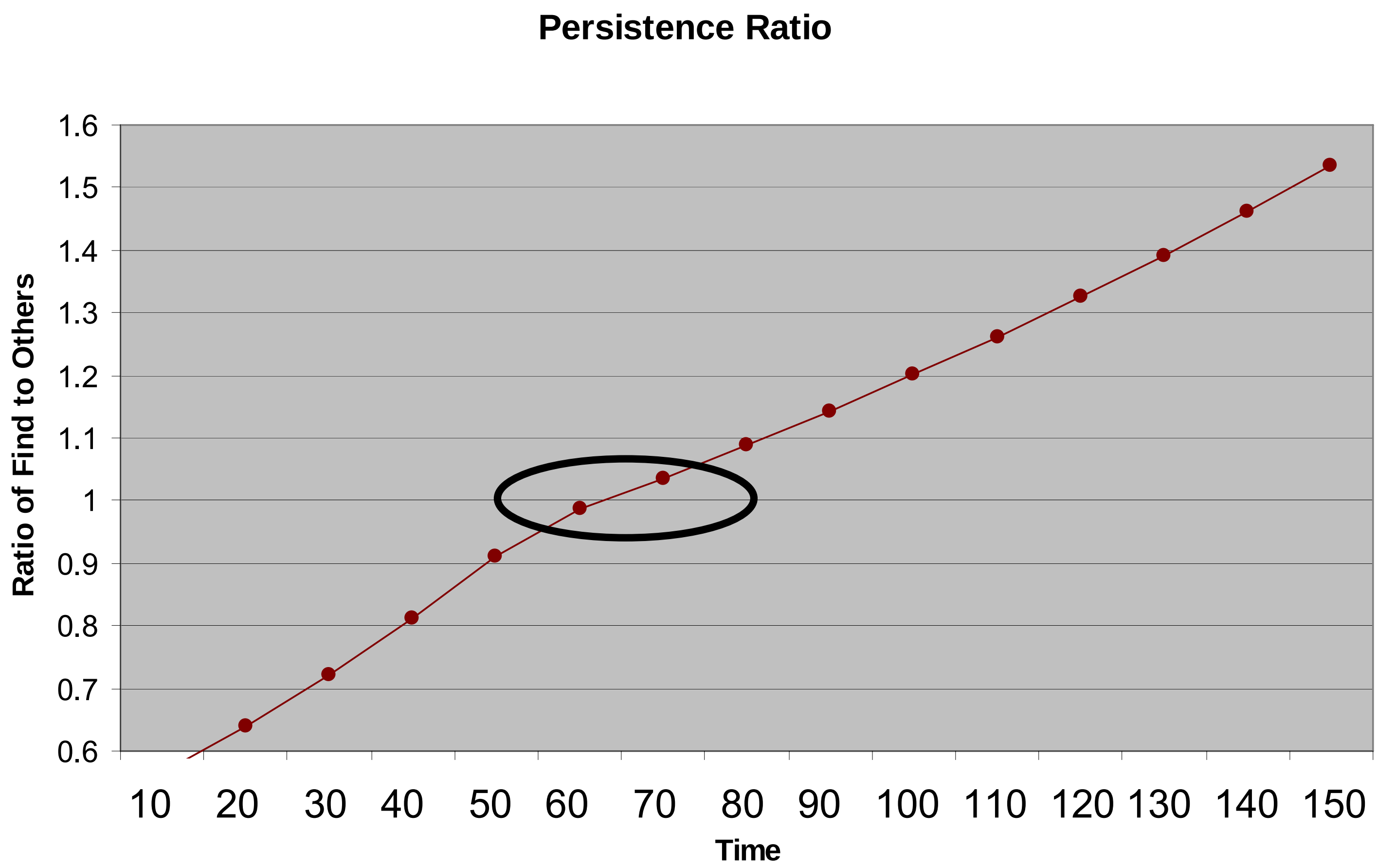
Given: Time varies (X axis) from 10 to 220 time units; θ_i = 20, 15, 10 respectively

Derivatives



Given: Time varies (X axis) from 10 to 150 time units; θ_i = 20, 15, 10 respectively

Persistent ISR Ratio



Minimum of Derivative (slope) of “Find” to other Derivatives.

Another Application: Solving for Window of Opportunity

- Find the minimum window of opportunity given a set of θ_is.
  - That is, if we know our θ_is and we want no less than a given *P(T),* we can solve *P(T)* equation for *T*.
  - For example, if θ_1 = 10 minutes, θ_2 = 15 minutes, and θ_3 = 5 minutes and we want no less than a P(FFF) = 0.80, we solve *P(T)* equation for *T* which yields *T* = 43.19 minutes.
  - This tells us that, given these θ_is, if we have a window of opportunity of less than 43.19 minutes, we have a *P(T)* < 0.80.

Now, we can perform cost-benefit trade studies

TELEDYNE
BROWN ENGINEERING, INC.
A Teledyne Technologies Company

Summary

- We've derived a closed form equation for *P(T)*
- We've derived closed form solutions for partial derivatives to study driving factors of *P(T)*
- We've looked at various applications
- We've developed a method for conducting cost-benefit studies and trade-offs.

TELEDYNE BROWN ENGINEERING, INC.
A Teledyne Technologies Company

EXAMPLES

Microsoft Excel Worksheet